Percolation Effects in Very High Energy Cosmic Rays

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Most QCD models of high energy collisions predict that the inelasticity K is an increasing function of the energy. We argue that, due to percolation of strings, this behaviour will change and, at $\sqrt{s} \simeq 10^4$ GeV, the inelasticity will start to decrease with the energy. This has straightforward consequences in high energy cosmic ray physics: 1) the relative depth of the shower maximum $\overline{X}$ grows faster with energy above the knee; 2) the energy measurements of ground array experiments at GZK energies could be overestimated.

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Most QCD-inspired models of multiparticle production predict, in hadron-hadron and nucleus-nucleus collisions at high energy, an increase with energy of the inelasticity parameter, $K \equiv 1 - x_F$, where x_F is the momentum fraction carried by the fastest particle: Multiple Scattering Model [1], Dual and string Models [2], and Minijet Model [3]. Recently, several papers appeared studying collective and non-linear QCD effects, based on the Colour Glass Condensate Model [4], Reggeon Calculus [5], and Strong Field string Model [6, 7]. These models predict large stopping power and a decrease of the momentum fraction carried by fast particles.

Two aspects are essential in the String Percolation Model [8, 9] that we use here: 1) At low energy (or density) leading valence strings are produced. As the energy increases, energy is drawn from the valence strings to produce, centrally in rapidity, sea strings. As in all the models mentioned above, the inelasticity increases with the energy. 2) At very high energy, above the percolation threshold, percolation leads to the formation of a large cluster of strings and to the production of faster particles. As a consequence the inelasticity starts to decrease with the energy.

While the relatively low energy regime, with K increasing, is similar to the models already mentioned, the higher energy regime, with decreasing K and the regeneration of the fast particles, is new and has some straightforward consequences in cosmic ray physics.

In the String Percolation Model [8, 9] for hadron-hadron collisions, at low energy, valence strings are formed, forward and backward in the centre-of-mass, along the collision axis, containing most of the collision energy. As the energy increases, additional sea strings, central in rapidity, are created, taking away part of the energy carried the valence strings. In the impact parameter plane all the strings look like disks, and we have to deal with a two dimension percolation problem.

The relevant parameter in percolation theory is the transverse density, η [10],

$$\eta \equiv (\frac{r}{\overline{R}})^2 \overline{N}_s \tag{1}$$

where r is the transverse radius of the string, $\overline{R}$ the effective radius of the interaction area. $\overline{N}_s$, the average number of strings, depends on the density (centrality) and on the energy. The strings may overlap in the interaction area, forming clusters of N strings. If $\eta \ll 1$, the average number of strings per cluster is $< N > \simeq 1$. If $\eta \gg 1, < N > \simeq \overline{N}_s$.

If $\overline{n}$ is the particle density for one string, $\overline{m}_T$ the average transverse mass produced from a single string, and there are $\overline{N}_s$ strings, one expects:

$$\frac{dn}{dy} = F(\eta)\overline{N}_s\overline{n} \quad \text{and} \quad < m_T >= \frac{1}{\sqrt{F(\eta)}}\overline{m}_T, \tag{2}$$

with a colour summation reduction factor [11, 12],

$$F(\eta) \equiv \sqrt{\frac{1 - e^{-\eta}}{\eta}}, \tag{3}$$

The particle density does not increase as fast as $\overline{N}_s$ (this corresponds to the saturation phenomenon [9]), and $< m_T >$ slowly increases with energy and density. These features are seen in data (see, for instance, [13]).

Following [14], let us consider proton-proton collisions and write for the invariant s,

$$s \equiv (P_1 + P_2)^2 \simeq 4P^2 = m^2 e^{\Delta Y}, \tag{4}$$

where $\vec{P}_{1,2}$ are the momenta of the protons, $P =\mid \vec{P}_1 \mid = \mid \vec{P}_2 \mid$, m is the proton mass, and ΔY the length of the rapidity "plateau". For a string made up of two partons with Feynman-x values x_- and x_+, we have, for the centre-of-mass energy of the two partons, $s_1 = x_- x_+ s$. Assuming for simplicity a symmetrical situation around the centre-of-mass, $x_- \simeq x_+ = \overline{x}$, the string centre-of-mass energy is $s_1 = \overline{x}^2 s$, and we can write the length of

the rapidity plateau for the string as:

$$\Delta y_1 = \Delta Y + 2 \ \ln \ \overline{x}. \tag{5}$$

If we write $\overline{x} = 2\overline{p}/\sqrt{s}$, where $\overline{p}$ is the absolute value of the momentum of the partons, we obtain:

$$\Delta y_1 = 2 \ \ln \ \frac{2\overline{p}}{m}. \tag{6}$$

We shall assume, for sea strings, that $\overline{p} =$ constant, which implies $\overline{x} \sim 1/\sqrt{s}$, in agreement with the rise of parton density at small x. Note that for valence strings, before energy degradation, $\overline{p} \sim P$, and the full phase space is occupied by the valence strings.

If strings overlap in the interaction region, and if $< N >$ is the average number of strings per cluster we have, generalising (5),

$$\Delta y_{<N>} = \Delta y_1 + 2 \ln < N > . \tag{7}$$

At low energy/density $< N > \simeq 1$ and only short strings are formed, not contributing to cosmic ray cascades. At high energy/density $< N > \simeq \overline{N}_s$, percolation occurs and the situation changes.

In percolation theory the average number $< N >$ of strings per cluster is related to the average area $< A >$, in units of r^2, occupied by a cluster [15],

$$< N > = < A > \frac{\eta}{1 - e^{-\eta}}, \tag{8}$$

with $< A >$ given by [16]:

$$< A > = f(\eta) \left[(\frac{\overline{R}}{r})^2 (1 - e^{-\eta}) - 1 \right] + 1, \tag{9}$$

where $f(\eta)$ is a percolation function,

$$f(\eta) = \left(1 + e^{-(\eta - \eta_c)/a} \right)^{-1}, \tag{10}$$

and $\eta_c \simeq 1.15$ is the transition point and $a \simeq 0,85$ is a parameter controlling the slope of the curve at the transition point, with $f(\eta)$ changing from 0 to 1 at $\eta \simeq \eta_c$. We note that when $\eta \rightarrow 0, < A > \simeq 1$, and when $\eta \rightarrow \infty, < A > \simeq (\overline{R}/r)^2$. This kind of parameterisation was tested in [15].

The energy, in the centre-of-mass, carried by the produced particles from sea strings is given by:

$$E_{CM} = \int_{-\frac{\Delta y_{<N>}}{2}}^{+\frac{\Delta y_{<N>}}{2}} < m_T > \ \cosh y \ \frac{dn}{dy} dy \tag{11}$$

and we obtain, making use of (2) and subtracting the 2 valence strings,

$$E_{CM} = \overline{m}_T \overline{n} \frac{1}{\sqrt{F(\eta)}} F(\eta) (\overline{N}_s - 2) \left[e^{\frac{\Delta y_{<N>}}{2}} - e^{-\frac{\Delta y_{<N>}}{2}} \right]. \tag{12}$$

If we now require that asymptotically all the energy is carried by the percolating strings,

$$E_{CM}(\sqrt{s} \rightarrow \infty) = \sqrt{s}, \tag{13}$$

we obtain, from (3), (7) and (12),

$$\overline{N}_s \underset{\sqrt{s} \rightarrow \infty}{\longrightarrow} s^{\lambda}, \ \text{with} \ \lambda = 2/7. \tag{14}$$

As $\overline{N}_s$ is proportional to the high energy bare Pomeron, the value of the intercept α_p is related to λ: $\alpha_p - 1 = \lambda$. This result is consistent with results from the Colour Glass Condensate Model [17].

In order to have an estimate of the inelasticity K we make use of the idea that, in the fragmentation of the string, produced particles are ordered in decreasing rapidity, and the fraction of momentum carried, relative to the momentum left, is always the same [18].

At small $\sqrt{s}$, when the valence strings carry all the energy, the fastest particle (F) is the leading particle (L) and $x_F = x_L = \alpha =$ const., with $0 < \alpha \leq 1$. When sea strings are produced, carrying an energy E_{CM}, we have,

$$x_L = \frac{2P_L}{\sqrt{s}} = \alpha \left(1 - \frac{E_{CM}}{\sqrt{s}} \right). \tag{15}$$

When the strings percolate, $E_{CM} \rightarrow 2 \mid \overline{P} \mid$ and for the fastest percolating particle (P) we have:

$$x_P = \alpha \frac{< N >}{\overline{N}_s} \frac{E_{CM}}{\sqrt{s}}. \tag{16}$$

As E_{CM} is an increasing function of $\sqrt{s}$, x_L decreases with the energy and x_P increases with energy. Thus,

$$K = \begin{cases} 1 - x_L, & \text{for } x_L > x_P \\ 1 - x_P, & \text{for } x_L < x_P \end{cases} \tag{17}$$

In order to implement the model (equations (2), (7), (12) and (15) to (17)), we have to establish the relation between $\overline{N}_s$ and η(eq. (1)), and to fix the parameters $r/\overline{R}$, m_T and Δy_1. At some low energy threshold, $\sqrt{s}_t \simeq$ 10 GeV, we have just the valence strings and $\overline{N}_s = 2$. At $\sqrt{s} \rightarrow \infty$, $\overline{N}_s \sim s^{\lambda}$ with λ given by equation (14). We then write:

$$\overline{N}_s = b + (2 - b)(\frac{s}{s_t})^{\lambda}. \tag{18}$$

where the parameter $b = 1.37$ was adjusted to agree with the data on dn/dy. The remaining parameters were fixed to reasonable values: $r/\overline{R} = 0.2$, $m_T = 0.78$ and $\Delta y_1 =$ 6. In this way, (13) was exactly satisfied.

In Fig. 1 the dn/dy data [19] is compared with the curve obtained from (2) and (18). With the parameterisation for $\overline{N}_s$ (18) we obtain that the critical density, η_c, occurs for $\sqrt{s} \simeq 10^4$ GeV.

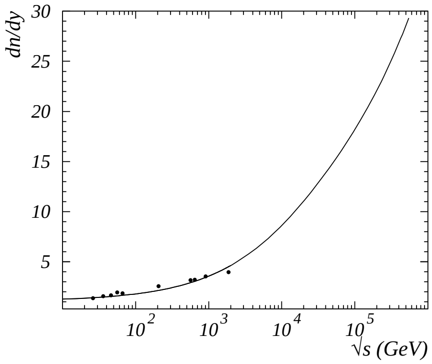

FIG. 1: Particle density as a function of $\sqrt{s}$. Data points are from [19]. The curve was obtained with (2,18) with $\sqrt{s_t} = 10$ GeV and $a = 1.37$.

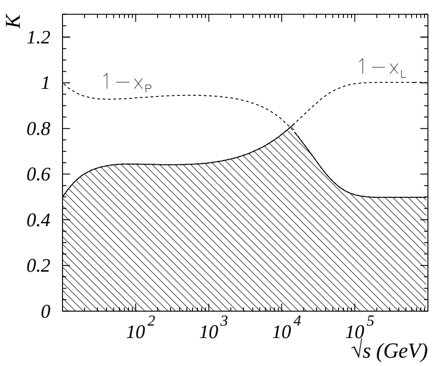

FIG. 2: The inelasticity parameter K as a function of $\sqrt{s}$, with $K = 1 - x_L$ at relatively low energies and $K = 1 - x_P$ at energies above the percolation threshold.

In Fig. 2 we show the $\sqrt{s}$ dependence of the inelasticity K (equation (17)). The behaviour of $1 - x_L$ and $1 - x_P$ is also shown in the Figure. From the combination of the two curves, K has a maximum at $\sqrt{s} \simeq 10^4$ GeV.

The development of showers, hadronic and electromagnetic, in cosmic ray physics, is critically dependent on the energy carried by the fast particles produced in the first hadronic collision, namely the leading particle. The shower evolves roughly exponentially, and reaches its maximum for a mean depth $\overline{X}$ (after the first collision).

Assuming now a simplified branching model, the relative shower maximum $\overline{X}$ can be expressed as:

$$\overline{X} = X_0 log_{10}\left[(1-K)E/E_0\right], \qquad (19)$$

where E is the laboratory energy ($E \simeq \frac{1}{2m}s$), K is the inelasticity (as defined in the present percolation model). $X_0 = 60$ g/cm^2 and $E_0 = 10^7$ eV are effective parameters related to the radiation length and to a low energy threshold for the shower branching, respectively. We have compared (19) to simulations [20] using hadronic interaction generators (Sibyll [21] and QGSjet [22], based respectively on the Dual Parton Model and Quark Gluon string Model), incorporated in the CORSIKA program [23]. The comparison is shown in Fig. 3: the parameterisation (19) is, at least for fixed energy, acceptable.

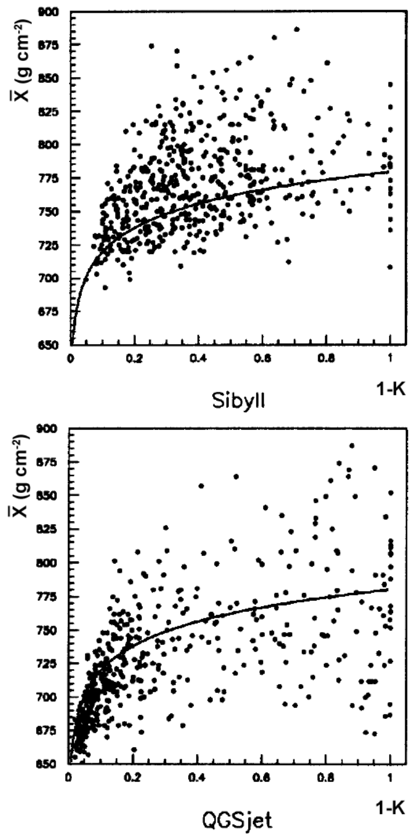

FIG. 3: The relative depth of the shower maximum as a function of the elasticity parameter $1 - K$ is shown for Sibyll and QGSjet simulations (taken from [20]) and compared to the parameterisation of equation (19).

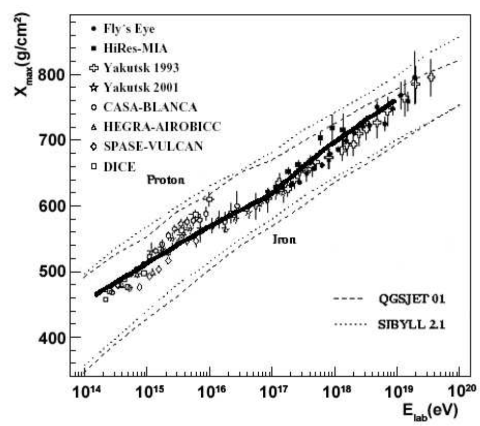

FIG. 4: The relative depth of the shower maximum as a function of the primary energy. The figure was adapted from [24], superimposing the result of the present percolation model (full line). Points are data and dashed lines show predictions of QGSJET and SIBYLL for protons and iron.

The dependence of $X_{max} = \overline{X} + X_0$ on the primary energy E is shown in Fig. 4. Near the percolation threshold ($E \sim 10^7$ GeV) there is a clear increase in the slope of $\overline{X}(E)$. In the present percolation model, the change in the slope of the $\overline{X}(E)$ curve can be qualitatively explained, in a natural way, through a change in the be-

haviour of the inelasticity, due to the effect of percolating sea strings above a certain energy. In contrast, this change in slope is usually explained by changing the fraction of heavy nuclei in cosmic rays (see for instance [24]): this fraction would be higher below the "kink" region (the region of fast growth of K), while above it the fraction of protons would rise (the region of decrease of K). In percolation models, a natural explanation arises without requiring a composition change. In fact, Fig. 4 shows that the percolation model line naturally goes from the model predictions for iron to those for protons as the energy increases. A quantitative description of the data is however beyond the scope of the present work, as it would imply a dedicated Monte Carlo simulation of proton-air interactions including percolation effects.

Furthermore, at very high energies a discrepancy between ground array experiments and fluorescence detectors is usually quoted [25]. We may note that the existence of percolation will basically only affect the energy measurement of ground arrays, which relies heavily on the Monte Carlo simulation of the shower development. In fact, including percolation K is lower and fast secondaries are expected. X_{max} will thus increase. This will lead, at the atmospheric depth of AGASA (taking into account that the acceptance is maximal for relatively inclined showers) both to a larger total number of particles and to a larger particle density in the region 600 m away from the shower core. This effect could partially explain the apparent contradiction between ground and fluorescence experiments at GZK energies.

Additional work covering in detail the general case of AB collisions and the treatment of the high energy behaviour of cross-sections is in preparation [27].

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